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Mark Scheme (Results)

Summer 2013

GCE Core Mathematics 4 (6666/01R)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- M marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- A marks: accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- B marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes:

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper
- $\sqsubset$ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

8. In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate's response may differ from the final mark scheme

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to x =

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to x =

2. Formula

Attempt to use correct formula (with values for *a*, *b* and *c*).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c, \quad q \neq 0,$ leading to x =...

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $(x^n \rightarrow x^{n-1})$

2. Integration

Power of at least one term increased by 1. $(x^n \rightarrow x^{n+1})$

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required.

Question Number	Scheme		Marks
1.	$\dfrac{5x+3}{(2x+1)(x+1)^2} \equiv \dfrac{A}{(2x+1)} + \dfrac{B}{(x+1)} + \dfrac{C}{(x+1)^2}$	At least one of "A" or "C" are correct.	B1
	$A = 2,\quad C = 2$	Breaks up their partial fraction correctly into three terms **and** both $"A" = 2$ and $"C" = 2$.	B1 **cso**
	$5x+3 \equiv A(x+1)^2 + B(2x+1)(x+1) + C(2x+1)$ $x = -1 \Rightarrow -2 = -C \Rightarrow C = 2$ $x = -\frac{1}{2} \Rightarrow -\frac{5}{2} + 3 = \frac{1}{4}A \Rightarrow \frac{1}{2} = \frac{1}{4}A \Rightarrow A = 2$	Writes down ***a correct identity*** and attempts to find the value of either one "A" or "B" or "C".	M1
	Either $x^2: 0 = A + 2B$, constant: $3 = A + B + C$ $x: 5 = 2A + 3B + 2C$		
	leading to $B = -1$	Correct value for "B" which is found using a correct identity and follows from their partial fraction decomposition.	A1 **cso**
			[4]
	So, $\dfrac{5x+3}{(2x+1)(x+1)^2} \equiv \dfrac{2}{(2x+1)} - \dfrac{1}{(x+1)} + \dfrac{2}{(x+1)^2}$		**4**

Notes for Question 1

BE CAREFUL! Candidates will assign ***their own*** "A, B and C" for this question.

B1: At least one of "A" or "C" are correct.

B1: Breaks up their partial fraction correctly into three terms **and** both $"A" = 2$ and $"C" = 2$.

M1: Writes down ***a correct identity*** (although this can be implied) and attempts to find the value of either one of "A" or "B" or "C".
This can be achieved by ***either*** substituting values into their identity ***or*** comparing coefficients and solving the resulting equations simultaneously.

A1: Correct value for "B" which is found using a correct identity and follows from their partial fraction decomposition.
Note: If a candidate does not give partial fraction decomposition then:
- the 2nd B1 mark can follow from a correct identity.
- the final A1 mark can be awarded for a correct "B" if a candidate goes writes out their partial fractions at the end.

Note: The correct partial fraction from no working scores B1B1M1A1.

Note: A number of candidates will start this problem by writing out the correct identity and then attempt to find "A" or "B" or "C". Therefore the B1 marks can be awarded from this method.

Question Number	Scheme		Marks
2.	$3^{x-1} + xy - y^2 + 5 = 0$		
		$3^{x-1} \rightarrow 3^{x-1}\ln 3$	B1 oe
	$\left\{\frac{dy}{dx} \times\right\}$ (ignore) $\quad 3^{x-1}\ln 3 + \left(y + x\frac{dy}{dx}\right) - 2y\frac{dy}{dx} = 0$	Differentiates implicitly to include either $\pm\lambda x\frac{dy}{dx}$ or $\pm ky\frac{dy}{dx}$.	M1*
		$xy \rightarrow +y + x\frac{dy}{dx}$	B1
		$\ldots + y + x\frac{dy}{dx} - 2y\frac{dy}{dx} = 0$	A1
	$\{(1, 3) \Rightarrow\}\ 3^{(1-1)}\ln 3 + 3 + (1)\frac{dy}{dx} - 2(3)\frac{dy}{dx} = 0$	Substitutes $x = 1,\ y = 3$ into their differentiated equation or expression.	dM1*
	$\ln 3 + 3 + \frac{dy}{dx} - 6\frac{dy}{dx} = 0 \Rightarrow 3 + \ln 3 = 5\frac{dy}{dx}$		
	$\frac{dy}{dx} = \frac{3 + \ln 3}{5}$		dM1*
	$\frac{dy}{dx} = \frac{1}{5}\left(\ln e^3 + \ln 3\right) = \frac{1}{5}\ln\left(3e^3\right)$	**Uses** $3 = \ln e^3$ to achieve $\frac{dy}{dx} = \frac{1}{5}\ln\left(3e^3\right)$	A1 **cso**
			[7] **7**

Notes for Question 2

B1: Correct differentiation of 3^{x-1}. I.e. $3^{x-1} \rightarrow 3^{x-1}\ln 3$ or $3^{x-1} = \frac{1}{3}\left(3^x\right) \rightarrow \frac{1}{3}\left(3^x\right)\ln 3$

or $3^{x-1} = e^{(x-1)\ln 3} \rightarrow \ln 3\, e^{(x-1)\ln 3}$ or $3^{x-1} = \frac{1}{3}\left(3^x\right) = \frac{1}{3}e^{x\ln 3} \rightarrow \frac{1}{3}(\ln 3)e^{x\ln 3}$

M1: Differentiates implicitly to include either $\pm\lambda x\frac{dy}{dx}$ or $\pm ky\frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx} = \right)$).

B1: $xy \rightarrow +y + x\frac{dy}{dx}$

1st A1: $\ldots + y + x\frac{dy}{dx} - 2y\frac{dy}{dx} = 0$ **Note:** The 1st A0 follows from an award of the 2nd B0.

Note: The "$= 0$" can be implied by rearrangement of their equation.

ie: $3^{x-1}\ln 3 + y + x\frac{dy}{dx} - 2y\frac{dy}{dx}$ leading to $3^{x-1}\ln 3 + y = 2y\frac{dy}{dx} - x\frac{dy}{dx}$ will get A1 (implied).

2nd M1: **Note:** This method mark is dependent upon the 1st M1* mark being awarded.
Substitutes $x = 1,\ y = 3$ into their differentiated equation or expression. Allow one slip.

3rd M1: **Note:** This method mark is dependent upon the 1st M1* mark being awarded.

Candidate has two differentiated terms in $\frac{dy}{dx}$ and rearranges to make $\frac{dy}{dx}$ the subject.

Note: It is possible to gain the 3rd M1 mark before the 2nd M1 mark.

Eg: Candidate may write $\frac{dy}{dx} = \frac{y + 3^{x-1}\ln 3}{2y - x}$ before substituting in $x = 1$ and $y = 3$

2nd A1: **cso.** Uses $3 = \ln e^3$ to achieve $\frac{dy}{dx} = \frac{1}{5}\ln\left(3e^3\right)$, $\left(= \frac{1}{\lambda}\ln\left(\mu e^3\right),\ \lambda = 5 \text{ and } \mu = 3\right)$

Note: $3 = \ln e^3$ needs to be seen in their proof.

Notes for Question 2 Continued

2.	**Alternative Method:** Multiplying both sides by 3 $3^{x-1} + xy - y^2 + 5 = 0$ $3^{x} + 3xy - 3y^2 + 15 = 0$		
		$3^x \rightarrow 3^x \ln 3$	B1
***Aliter* Way 2**	$\left\{\frac{dy}{dx} \times\right\}$ $3^x \ln 3 + \left(3y + 3x\frac{dy}{dx}\right) - 6y\frac{dy}{dx} = 0$ (ignore)	Differentiates implicitly to include either $\pm\lambda x\frac{dy}{dx}$ or $\pm ky\frac{dy}{dx}$.	M1*
		$3xy \rightarrow +3y + 3x\frac{dy}{dx}$	B1
		$... + 3y + 3x\frac{dy}{dx} - 6y\frac{dy}{dx} = 0$	A1
	$\{(1, 3) \Rightarrow\}$ $3^1 \ln 3 + 3(3) + (3)(1)\frac{dy}{dx} - 6(3)\frac{dy}{dx} = 0$	Substitutes $x = 1,\ y = 3$ into their differentiated equation or expression.	dM1*
	$3\ln 3 + 9 + 3\frac{dy}{dx} - 18\frac{dy}{dx} = 0 \Rightarrow 9 + 3\ln 3 = 15\frac{dy}{dx}$		
	$\frac{dy}{dx} = \frac{9 + 3\ln 3}{15}\left\{= \frac{3 + \ln 3}{5}\right\}$		dM1*
	$\frac{dy}{dx} = \frac{1}{5}\left(\ln e^3 + \ln 3\right)$		
	$\frac{dy}{dx} = \frac{1}{5}\left(\ln e^3 + \ln 3\right) = \frac{1}{5}\ln\left(3e^3\right)$	**Uses** $3 = \ln e^3$ to achieve $\frac{dy}{dx} = \frac{1}{5}\ln\left(3e^3\right)$	A1 **cso**
			[7] **7**
	NOTE: Only apply this scheme if the candidate has multiplied both sides of their equation by 3. **NOTE:** For reference, $\frac{dy}{dx} = \frac{3y + 3^x \ln 3}{6y - 3x}$ **NOTE:** If the candidate applies this method then $3xy \rightarrow +3y + 3x\frac{dy}{dx}$ must be seen for the 2nd B1 mark.		

Question Number	Scheme		Marks
3.	$\displaystyle\int_0^4 \frac{1}{2+\sqrt{(2x+1)}}\,\mathrm{d}x$, $u = 2+\sqrt{(2x+1)}$		
	$\dfrac{\mathrm{d}u}{\mathrm{d}x} = (2x+1)^{-\frac{1}{2}}$ or $\dfrac{\mathrm{d}x}{\mathrm{d}u} = u-2$	Either $\dfrac{\mathrm{d}u}{\mathrm{d}x} = \pm K(2x+1)^{-\frac{1}{2}}$ or $\dfrac{\mathrm{d}x}{\mathrm{d}u} = \pm\lambda(u-2)$	M1
		Either $\dfrac{\mathrm{d}u}{\mathrm{d}x} = (2x+1)^{-\frac{1}{2}}$ or $\dfrac{\mathrm{d}x}{\mathrm{d}u} = (u-2)$	A1
	$\left\{\displaystyle\int \frac{1}{2+\sqrt{(2x+1)}}\,\mathrm{d}x\right\} = \displaystyle\int \frac{1}{u}(u-2)\,\mathrm{d}u$	Correct substitution (Ignore integral sign and $\mathrm{d}u$).	A1
	$= \displaystyle\int \left(1-\frac{2}{u}\right)\mathrm{d}u$	An attempt to divide each term by u.	dM1
		$\pm Au \pm B\ln u$	ddM1
	$= u - 2\ln u$	$u - 2\ln u$	A1 ft
	$\left\{\text{So } \left[u-2\ln u\right]_3^5\right\} = (5-2\ln 5)-(3-2\ln 3)$	Applies limits of 5 and 3 in u or 4 and 0 in x in their integrated function and subtracts the correct way round.	M1
	$= 2+2\ln\left(\dfrac{3}{5}\right)$	$2+2\ln\left(\dfrac{3}{5}\right)$	A1 **cao cso**
			[8] **8**

Notes for Question 3

M1: Also allow $\mathrm{d}u = \pm\lambda\dfrac{1}{(u-2)}\mathrm{d}x$ or $(u-2)\mathrm{d}u = \pm\lambda\,\mathrm{d}x$

Note: The expressions must contain $\mathrm{d}u$ and $\mathrm{d}x$. They can be simplified or un-simplified.

A1: Also allow $\mathrm{d}u = \dfrac{1}{(u-2)}\mathrm{d}x$ or $(u-2)\mathrm{d}u = \pm\lambda\,\mathrm{d}x$

Note: The expressions must contain $\mathrm{d}u$ and $\mathrm{d}x$. They can be simplified or un-simplified.

A1: $\displaystyle\int \frac{1}{u}(u-2)\,\mathrm{d}u$. (Ignore integral sign and $\mathrm{d}u$).

dM1: An attempt to divide each term by u.
Note that this mark is dependent on the previous M1 mark being awarded.
Note that this mark can be implied by later working.

ddM1: $\pm Au \pm B\ln u$, $A \neq 0$, $B \neq 0$
Note that this mark is dependent on the two previous M1 marks being awarded.

A1ft: $u - 2\ln u$ or $\pm Au \pm B\ln u$ being correctly followed through, $A \neq 0$, $B \neq 0$

M1: Applies limits of 5 and 3 in u or 4 and 0 in x in their integrated function and subtracts the correct way round.

A1: **cso** and **cao**. $2+2\ln\left(\dfrac{3}{5}\right)$ or $2+2\ln(0.6)$, $\left(= A+2\ln B, \text{ so } A=2, B=\dfrac{3}{5}\right)$

Note: $2-2\ln\left(\dfrac{3}{5}\right)$ is A0.

Notes for Question 3 Continued	
3. ctd	**Note:** $\int \frac{1}{u}(u-2)\,\mathrm{d}u = u - 2\ln u$ with no working is 2nd M1, 3rd M1, 3rd A1. but **Note:** $\int \frac{1}{u}(u-2)\,\mathrm{d}u = (u-2)\ln u$ with no working is 2nd M0, 3rd M0, 3rd A0.

Question Number	Scheme		Marks
4. (a)	$\left\{\sqrt[3]{(8-9x)}\right\} = (8-9x)^{\frac{1}{3}}$	Power of $\frac{1}{3}$	M1
	$= \underline{(8)^{\frac{1}{3}}}\left(1-\frac{9x}{8}\right)^{\frac{1}{3}} = \underline{2}\left(1-\frac{9x}{8}\right)^{\frac{1}{3}}$	$\underline{(8)^{\frac{1}{3}}}$ or $\underline{2}$	$\underline{\text{B1}}$
	$= \{2\}\left[1+\left(\frac{1}{3}\right)(kx)+\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(kx)^2+\frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(kx)^3+...\right]$	see notes	M1 A1
	$= \{2\}\left[\underline{1+\left(\frac{1}{3}\right)\left(\frac{-9x}{8}\right)+\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}\left(\frac{-9x}{8}\right)^2+\frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}\left(\frac{-9x}{8}\right)^3+...}\right]$		
	$= 2\left[1-\frac{3}{8}x;-\frac{9}{64}x^2-\frac{45}{512}x^3+...\right]$	See notes below!	
	$= 2-\frac{3}{4}x;-\frac{9}{32}x^2-\frac{45}{256}x^3+...$		A1; A1
			[6]
(b)	$\left\{\sqrt[3]{7100} = 10\sqrt[3]{71} = 10\sqrt[3]{(8-9x)},\right\}$ so $x = 0.1$	Writes down or uses $x = 0.1$	B1
	When $x = 0.1$, $\sqrt[3]{(8-9x)} \approx 2-\frac{3}{4}(0.1)-\frac{9}{32}(0.1)^2-\frac{45}{256}(0.1)^3+...$ $= 2 - 0.075 - 0.0028125 - 0.00017578125$ $= 1.922011719$		M1
	So, $\sqrt[3]{7100} = 19.220117919... = \underline{19.2201}$ (4 dp)	19.2201 **cso**	A1 **cao**
			[3] **9**

Notes for Question 4

(a)	**M1:** Writes or uses $\frac{1}{3}$. This mark can be implied by a constant term of $8^{\frac{1}{3}}$ or 2. $\underline{\textbf{B1}}$: $\underline{(8)^{\frac{1}{3}}}$ or $\underline{2}$ outside brackets or $\underline{2}$ as candidate's constant term in their binomial expansion. **M1:** Expands $(...+kx)^{\frac{1}{3}}$ to give any 2 terms out of 4 terms simplified or un-simplified, Eg: $1+\left(\frac{1}{3}\right)(kx)$ or $\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(kx)^2+\frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(kx)^3$ or $1+ ...... +\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(kx)^2$ or $\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(kx)^2+\frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(kx)^3$ **where** $k \neq 1$ are fine for M1. **A1:** A correct simplified or un-simplified $1+\left(\frac{1}{3}\right)(kx)+\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(kx)^2+\frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(kx)^3$ expansion with consistent (kx). Note that (kx) must be consistent (on the RHS, not necessarily the LHS) in a candidate's expansion. **Note** that $k \neq 1$**.** You would award B1M1A0 for $2\left[\underline{1+\left(\frac{1}{3}\right)\left(\frac{-9x}{8}\right)+\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(-9x)^2+\frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}\left(\frac{-9x}{8}\right)^3+...}\right]$ because (kx) is not consistent.

Notes for Question 4 Continued

4. (a) ctd

"Incorrect bracketing" $=\{2\}\left[\underline{1+\left(\frac{1}{3}\right)\left(\frac{-9x}{8}\right)+\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}\left(\frac{-9x^2}{8}\right)+\frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}\left(\frac{-9x^3}{8}\right)+\ldots}\right]$

is M1A0 unless recovered.

A1: For $2-\frac{3}{4}x$ **(simplified please)** or also allow $2-0.75x$.

Allow Special Case A1A0 for either SC: $=2\left[1-\frac{3}{8}x;\ \ldots\right]$ **or SC**: $K\left[1-\frac{3}{8}x-\frac{9}{64}x^2-\frac{45}{512}x^3+\ldots\right]$

(where K can be 1 or omitted), with each term in the $[\ldots\ldots\ldots]$ either a simplified fraction or a decimal.

A1: Accept only $-\frac{9}{32}x^2-\frac{45}{256}x^3$ or $-0.28125x^2-0.17578125x^3$

Candidates who write $=2\left[\underline{1+\left(\frac{1}{3}\right)\left(\frac{9x}{8}\right)+\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}\left(\frac{9x}{8}\right)^2+\frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}\left(\frac{9x}{8}\right)^3+\ldots}\right]$ where $k=\frac{9}{8}$

and not $-\frac{9}{8}$ and achieve $2+\frac{3}{4}x;-\frac{9}{32}x^2+\frac{45}{256}x^3+\ldots$ will get B1M1A1A0A0.

Note for final two marks:

$2\left[1-\frac{3}{8}x;-\frac{9}{64}x^2-\frac{45}{512}x^3+\ldots\right]=2+\frac{3}{4}x-\frac{9}{32}x^2-\frac{45}{256}x^3+\ldots$ scores final A0A1.

$2\left[1-\frac{3}{8}x;-\frac{9}{64}x^2-\frac{45}{512}x^3+\ldots\right]=2-\frac{3}{4}-\frac{9}{32}x^2-\frac{45}{256}x^3+\ldots$ scores final A0A1

Alternative method: Candidates can apply an alternative form of the binomial expansion.

$$\left\{\sqrt[3]{(8-9x)}\right\}=(8-9x)^{\frac{1}{3}}=(8)^{\frac{1}{3}}+\left(\frac{1}{3}\right)(8)^{-\frac{2}{3}}(-9x)+\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(8)^{-\frac{5}{3}}(-9x)^2+\frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(8)^{-\frac{8}{3}}(-9x)^3$$

B1: $(8)^{\frac{1}{3}}$ or 2

M1: Any two of four (un-simplified or simplified) terms correct.

A1: All four (un-simplified or simplified) terms correct.

A1: $2-\frac{3}{4}x$

A1: $-\frac{9}{32}x^2-\frac{45}{256}x^3$

Note: The terms in C need to be evaluated,

so ${}^{\frac{1}{3}}C_0(8)^{\frac{1}{3}}+{}^{\frac{1}{3}}C_1(8)^{-\frac{2}{3}}(-9x)+{}^{\frac{1}{3}}C_2(8)^{-\frac{5}{3}}(-9x)^2+{}^{\frac{1}{3}}C_3(8)^{-\frac{8}{3}}(-9x)^3$ without further working is B0M0A0.

(b)

B1: Writes down or uses $x=0.1$

M1: Substitutes their x, where $|x|<\frac{8}{9}$ into at least two terms of their binomial expansion.

A1: 19.2201 **cao**

Be Careful! The binomial answer is 19.22011719

and the calculated $\sqrt[3]{7100}$ is 19.21997343... which is 19.2200 to 4 decimal places.

Question Number	Scheme		Marks
5. (a)	$6.248046798... = 6.248$ (3dp)	6.248 or awrt 6.248	B1
			[1]
(b)	Area $\approx \frac{1}{2} \times 2\,; \times \underline{\left[3 + 2\left(7.107 + 7.218 + \text{their } 6.248\right) + 5.223\right]}$		B1; <u>M1</u>
	$= 49.369 = 49.37$ (2 dp)	49.37 or awrt 49.37	A1
			[3]
(c)	$\left\{\int (4t\mathrm{e}^{-\frac{1}{3}t} + 3)\,\mathrm{d}t\right\} = -12t\mathrm{e}^{-\frac{1}{3}t} - \int -12\mathrm{e}^{-\frac{1}{3}t}\{\mathrm{d}t\}$	$\pm At\mathrm{e}^{-\frac{1}{3}t} \pm B\int \mathrm{e}^{-\frac{1}{3}t}\{\mathrm{d}t\}$, $A \neq 0, B \neq 0$	M1
		See notes.	A1
	$+ 3t$	$3 \to 3t$	B1
	$= -12t\mathrm{e}^{-\frac{1}{3}t} - 36\mathrm{e}^{-\frac{1}{3}t}\,\{+3t\}$	$-12t\mathrm{e}^{-\frac{1}{3}t} - 36\mathrm{e}^{-\frac{1}{3}t}$	A1
	$\left[-12t\mathrm{e}^{-\frac{1}{3}t} - 36\mathrm{e}^{-\frac{1}{3}t} + 3t\right]_0^8 =$		
	$= \left(-12(8)\mathrm{e}^{-\frac{1}{3}(8)} - 36\mathrm{e}^{-\frac{1}{3}(8)} + 3(8)\right) - \left(-12(0)\mathrm{e}^{-\frac{1}{3}(0)} - 36\mathrm{e}^{-\frac{1}{3}(0)} + 3(0)\right)$	Substitutes limits of 8 and 0 into an integrated function of the form of either $\pm\lambda t\mathrm{e}^{-\frac{1}{3}t} \pm \mu\mathrm{e}^{-\frac{1}{3}t}$ or $\pm\lambda t\mathrm{e}^{-\frac{1}{3}t} \pm \mu\mathrm{e}^{-\frac{1}{3}t} + Bt$ and subtracts the correct way round.	dM1
	$= \left(-96\mathrm{e}^{-\frac{8}{3}} - 36\mathrm{e}^{-\frac{8}{3}} + 24\right) - (0 - 36 + 0)$		
	$= 60 - 132\mathrm{e}^{-\frac{8}{3}}$	$60 - 132\mathrm{e}^{-\frac{8}{3}}$	A1
			[6]
(d)	Difference $= \left\|60 - 132\mathrm{e}^{-\frac{8}{3}} - 49.37\right\| = 1.458184439... = 1.46$ (2 dp)	1.46 or awrt 1.46	B1
			[1]
			11

Notes for Question 5

(a)	**B1:** 6.248 or awrt 6.248. Look for this on the table or in the candidate's working.
(b)	**B1**: Outside brackets $\frac{1}{2} \times 2$ or 1
	M1: <u>For structure of trapezium rule</u> $[\ \ldots\ldots\ldots\ldots\ ]$. Allow one miscopy of their values.
	A1: 49.37 or anything that rounds to 49.37 **Note:** It can be possible to award : (a) B0 (b) B1M1A1 (awrt 49.37) **<u>Note:</u>** Working must be seen to demonstrate the use of the trapezium rule. **<u>Note</u>**: actual area is 50.828… <u>Bracketing mistake</u>: **Unless the final answer implies that the calculation has been done correctly,** Award B1M0A0 for $1 + 3 + 2\left(7.107 + 7.218 + \text{their } 6.248\right) + 5.223$ (nb: answer of 50.369).

Notes for Question 5 Continued	
5. (b) ctd	<u>*Alternative method for part (b): Adding individual trapezia*</u> Area $\approx 2\times\left[\frac{3+7.107}{2}+\frac{7.107+7.218}{2}+\frac{7.218+6.248}{2}+\frac{6.248+5.223}{2}\right] = 49.369$ **B1:** 2 and a divisor of 2 on all terms inside brackets. **M1:** First and last ordinates once and two of the middle ordinates twice inside brackets ignoring the 2. **A1:** anything that rounds to 49.37
(c)	**M1:** For $4t\mathrm{e}^{-\frac{1}{3}t} \rightarrow \pm At\mathrm{e}^{-\frac{1}{3}t} \pm B\int \mathrm{e}^{-\frac{1}{3}t}\{\mathrm{d}t\}$, $A \neq 0, B \neq 0$ **A1:** For $t\mathrm{e}^{-\frac{1}{3}t} \rightarrow \left(-3t\mathrm{e}^{-\frac{1}{3}t} - \int -3\mathrm{e}^{-\frac{1}{3}t}\right)$ (some candidates lose the 4 and this is fine for the first A1 mark). or $4t\mathrm{e}^{-\frac{1}{3}t} \rightarrow 4\left(-3t\mathrm{e}^{-\frac{1}{3}t} - \int -3\mathrm{e}^{-\frac{1}{3}t}\right)$ or $-12t\mathrm{e}^{-\frac{1}{3}t} - \int -12\mathrm{e}^{-\frac{1}{3}t}$ or $12\left(-t\mathrm{e}^{-\frac{1}{3}t} - \int -\mathrm{e}^{-\frac{1}{3}t}\right)$ These results can be implied. They can be simplified or un-simplified. **B1:** $3 \rightarrow 3t$ or $3 \rightarrow 3x$ (bod) . **Note:** Award B0 for 3 integrating to $12t$ (implied), which is a common error when taking out a factor of 4. **Be careful** some candidates will factorise out 4 and have $4\left(....+\frac{3}{4}\right) \rightarrow 4\left(....+\frac{3}{4}t\right)$ which would then be fine for B1. **Note:** Allow B1 for $\int_0^8 3\,\mathrm{d}t = 24$ **A1:** For correct integration of $4t\mathrm{e}^{-\frac{1}{3}t}$ to give $-12t\mathrm{e}^{-\frac{1}{3}t} - 36\mathrm{e}^{-\frac{1}{3}t}$ or $4\left(-3t\mathrm{e}^{-\frac{1}{3}t} - 9\mathrm{e}^{-\frac{1}{3}t}\right)$ or equivalent. This can be simplified or un-simplified. **dM1:** Substitutes limits of 8 and 0 into an integrated function of the form of either $\pm\lambda t\mathrm{e}^{-\frac{1}{3}t} \pm \mu\mathrm{e}^{-\frac{1}{3}t}$ or $\pm\lambda t\mathrm{e}^{-\frac{1}{3}t} \pm \mu\mathrm{e}^{-\frac{1}{3}t} + Bt$ and subtracts the correct way round. **Note:** Evidence of a proper consideration of the limit of 0 (as detailed in the scheme) is needed for dM1. So, just subtracting zero is M0. **A1:** An exact answer of $60 - 132\mathrm{e}^{-\frac{8}{3}}$. A decimal answer of 50.82818444... without a correct answer is A0. **Note:** A decimal answer of 50.82818444... without a correct exact answer is A0. **Note:** If a candidate gains M1A1B1A1 and then writes down 50.8 or awrt 50.8 with no method for substituting limits of 8 and 0, then award the final M1A0. **IMPORTANT:** that is fine for candidates to work in terms of x rather than t in part (c). **Note:** The "$3t$" is needed for B1 and the final A1 mark.
(d)	**B1:** 1.46 or awrt 1.46 or -1.46 or awrt -1.46. Candidates may give correct decimal answers of 1.458184439... or 1.459184439... **Note:** You can award this mark whether or not the candidate has answered part (c) correctly.

Question Number	Scheme		Marks
6.	$l: \mathbf{r} = \begin{pmatrix} a \\ b \\ 10 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ c \\ -1 \end{pmatrix}, \quad \overrightarrow{OA} = \begin{pmatrix} 21 \\ -17 \\ 6 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 25 \\ -14 \\ 18 \end{pmatrix}$		
(a)	A is on l, so $\begin{pmatrix} 21 \\ -17 \\ 6 \end{pmatrix} = \begin{pmatrix} a \\ b \\ 10 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ c \\ -1 \end{pmatrix}$		
	$\{\mathbf{k}: \ 10 - \lambda = 6 \Rightarrow\} \ \lambda = 4$	$\lambda = 4$	B1
	$\{\mathbf{i}: \ a + 6\lambda = 21 \Rightarrow\} \ a + 6(4) = 21$	Substitutes their value of λ into $a + 6\lambda = 21$	M1
	$a = -3$	$a = -3$	A1 **cao**
			[3]
(b)	$\{\overrightarrow{AB}\} = \begin{pmatrix} 25 \\ -14 \\ 18 \end{pmatrix} - \begin{pmatrix} 21 \\ -17 \\ 6 \end{pmatrix}$ | $\{\overrightarrow{BA}\} = \begin{pmatrix} 21 \\ -17 \\ 6 \end{pmatrix} - \begin{pmatrix} 25 \\ -14 \\ 18 \end{pmatrix}$	Finds the difference between $\overrightarrow{OA}$ and $\overrightarrow{OB}$. Ignore labelling.	M1
	$\{\overrightarrow{AB}\} = \begin{pmatrix} 4 \\ 3 \\ 12 \end{pmatrix}$ | $\{\overrightarrow{BA}\} = \begin{pmatrix} -4 \\ -3 \\ -12 \end{pmatrix}$		
	$\{\overrightarrow{AB} \perp l \Rightarrow \overrightarrow{AB} \bullet \mathbf{d} = 0\} \Rightarrow \begin{pmatrix} 4 \\ 3 \\ 12 \end{pmatrix} \bullet \begin{pmatrix} 6 \\ c \\ -1 \end{pmatrix} = 24 + 3c - 12 = 0; \Rightarrow c = -4$	See notes.	M1; A1 ft
	$\{\mathbf{j}: \ b + c\lambda = -17 \Rightarrow\} \ b + (-4)(4) = -17 \ ; \Rightarrow \ b = -1$	See notes.	ddM1; A1 **cso cao**
			[5]
(c)	$\lvert AB \rvert = \sqrt{4^2 + 3^2 + 12^2}$ or $\lvert AB \rvert = \sqrt{(-4)^2 + (-3)^2 + (-12)^2}$	See notes.	M1
	So, $\lvert AB \rvert = 13$		A1 **cao**
			[2]
(d)	$\overrightarrow{OB'} \ \{= \overrightarrow{OA} + \overrightarrow{BA}\} = \begin{pmatrix} 21 \\ -17 \\ 6 \end{pmatrix} + \begin{pmatrix} -4 \\ -3 \\ -12 \end{pmatrix}; = \begin{pmatrix} 17 \\ -20 \\ -6 \end{pmatrix}$	See notes for alternative methods.	M1;A1 **cao**
			[2] **12**

Notes for Question 6	
(a)	**B1:** $\lambda = 4$ seen or implied. **M1:** Substitutes their value of λ into $a + 6\lambda = 21$ **A1:** $a = -3$. Note: Award B1M1A1 if the candidate states $a = -3$ from no working. ***<u>Alternative Method Using Simultaneous equations for part (a).</u>*** **B1:** For $60 - 6\lambda = 36$ **M1:** $60 - 6\lambda = 36$ and $a + 6\lambda = 21$ solved simultaneously to give $a = ...$ **A1:** $a = -3$, **cao**.

Notes for Question 6 Continued

6. (b) ctd

M1: Finds the difference between $\overrightarrow{OA}$ and $\overrightarrow{OB}$. Ignore labelling.
If no "subtraction" seen, you can award M1 for 2 out of 3 correct components of the difference.

M1: ***Applies*** the formula $\overrightarrow{AB} \bullet \begin{pmatrix} 6 \\ c \\ -1 \end{pmatrix}$ or $\overrightarrow{BA} \bullet \begin{pmatrix} 6 \\ c \\ -1 \end{pmatrix}$ correctly to give a linear equation in c which is set equal to zero. **Note:** The dot product can also be with $\pm k \begin{pmatrix} 6 \\ c \\ -1 \end{pmatrix}$.

A1ft: $c = -4$ **or** for finding a correct follow through c.

ddM1: Substitutes their value of λ and their value of c into $b + c\lambda = -17$
Note that this mark is dependent on the two previous method marks being awarded.

A1: $b = -1$

(c)

M1: An attempt to apply a three term Pythagoras in order to find $|AB|$,
so taking the square root is required here.

A1: 13 **cao**

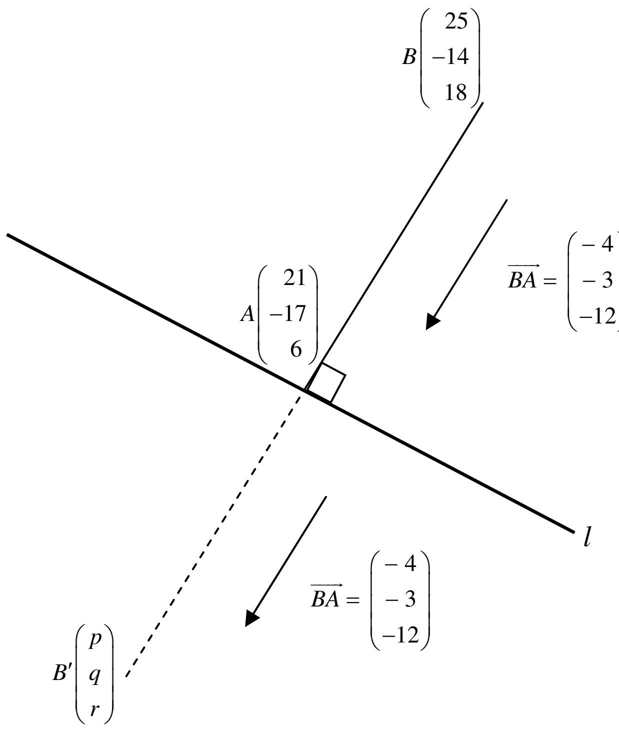
Note: Don't recover work for part (b) in part (c).

(d)

M1: For a full ***applied*** method of finding the coordinates of B'.
Note: You can give M1 for 2 out of 3 correct components of B'.

A1: For either $\begin{pmatrix} 17 \\ -20 \\ -6 \end{pmatrix}$ or $17\mathbf{i} - 20\mathbf{j} - 6\mathbf{k}$ or $(17, -20, -6)$ **cao**.

Helpful diagram!



Notes for Question 6 Continued		
	Acceptable Methods for the Method mark in part (d)	
Way 1	$\overrightarrow{OB'}\left\{=\overrightarrow{OA}+\overrightarrow{BA}\right\}=\begin{pmatrix}21\\-17\\6\end{pmatrix}+\begin{pmatrix}-4\\-3\\-12\end{pmatrix}$ (using their $\overrightarrow{BA}$)	
Way 2	$\overrightarrow{OB'}\left\{=\overrightarrow{OA}-\overrightarrow{AB}\right\}=\begin{pmatrix}21\\-17\\6\end{pmatrix}-\begin{pmatrix}4\\3\\12\end{pmatrix}$ (using their $\overrightarrow{AB}$)	
Way 3	$\overrightarrow{OB'}\left\{=\overrightarrow{OB}+2\overrightarrow{BA}\right\}=\begin{pmatrix}25\\-14\\18\end{pmatrix}+2\begin{pmatrix}-4\\-3\\-12\end{pmatrix}$ (using their $\overrightarrow{BA}$)	
Way 4	$\overrightarrow{OB'}\left\{=\overrightarrow{OB}-2\overrightarrow{AB}\right\}=\begin{pmatrix}25\\-14\\18\end{pmatrix}-2\begin{pmatrix}4\\3\\12\end{pmatrix}$ (using their $\overrightarrow{AB}$)	
Way 5	$\begin{pmatrix}25\\-14\\18\end{pmatrix}\to\begin{pmatrix}\text{Minus 4}\\\text{Minus 3}\\\text{Minus 12}\end{pmatrix}\to\begin{pmatrix}21\\-17\\6\end{pmatrix}\to\begin{pmatrix}\text{Minus 4}\\\text{Minus 3}\\\text{Minus 12}\end{pmatrix}\left\{\to\begin{pmatrix}17\\-20\\-6\end{pmatrix}\right\}$, so $\overrightarrow{OA}$ + their $\overrightarrow{BA}$	
Way 6	$\overrightarrow{OB'}\left\{=2\overrightarrow{OA}-\overrightarrow{OB}\right\}=2\begin{pmatrix}21\\-17\\6\end{pmatrix}-\begin{pmatrix}25\\-14\\18\end{pmatrix}$	
Way 7	$\overrightarrow{OB}=25\mathbf{i}-14\mathbf{j}+18\mathbf{k}$, $\overrightarrow{OA}=21\mathbf{i}-17\mathbf{j}+6\mathbf{k}$ and $\overrightarrow{OB'}=p\mathbf{i}+q\mathbf{j}+r\mathbf{k}$, $(21,-17,6)=\left(\frac{25+p}{2},\frac{-14+q}{2},\frac{18+r}{2}\right)$ $p=21(2)-25=17$ $q=-17(2)+14=-20$ $r=6(2)-18=-6$	**M1:** Writing down any two equations correctly and an attempt to find at least two of p, q or r.

Question Number	Scheme		Marks
7.	$x = 27\sec^3 t,\quad y = 3\tan t,\quad 0 \leqslant t \leqslant \frac{\pi}{3}$		
(a)	$\frac{dx}{dt} = 81\sec^2 t \sec t \tan t,\quad \frac{dy}{dt} = 3\sec^2 t$	At least one of $\frac{dx}{dt}$ or $\frac{dy}{dt}$ correct.	B1
		Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are correct.	B1
	$\frac{dy}{dx} = \frac{3\sec^2 t}{81\sec^3 t \tan t} \left\{= \frac{1}{27\sec t \tan t} = \frac{\cos t}{27\tan t} = \frac{\cos^2 t}{27\sin t}\right\}$	Applies their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$	M1;
	At $t = \frac{\pi}{6}$, $\frac{dy}{dx} = \frac{3\sec^2\left(\frac{\pi}{6}\right)}{81\sec^3\left(\frac{\pi}{6}\right)\tan\left(\frac{\pi}{6}\right)} = \frac{4}{72} \left\{= \frac{3}{54} = \frac{1}{18}\right\}$	$\frac{4}{72}$	A1 **cao cso**
			[4]
(b)	$\{1 + \tan^2 t = \sec^2 t\} \Rightarrow 1 + \left(\frac{y}{3}\right)^2 = \left(\sqrt[3]{\left(\frac{x}{27}\right)}\right)^2 = \left(\frac{x}{27}\right)^{\frac{2}{3}}$		M1
	$\Rightarrow 1 + \frac{y^2}{9} = \frac{x^{\frac{2}{3}}}{9} \Rightarrow 9 + y^2 = x^{\frac{2}{3}} \Rightarrow y = \left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}}$ *		A1 * **cso**
	$a = 27$ and $b = 216$ or $27 \leqslant x \leqslant 216$	$a = 27$ and $b = 216$	B1
			[3]
(c)	$V = \pi\int_{27}^{125}\left(\left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}}\right)^2 dx$ or $\pi\int_{27}^{125}\left(x^{\frac{2}{3}} - 9\right)dx$	For $\pi\int\left(\left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}}\right)^2$ or $\pi\int\left(x^{\frac{2}{3}} - 9\right)$ Ignore limits and dx. Can be implied.	B1
	$= \{\pi\}\left[\frac{3}{5}x^{\frac{5}{3}} - 9x\right]_{27}^{125}$	Either $\pm Ax^{\frac{5}{3}} \pm Bx$ or $\frac{3}{5}x^{\frac{5}{3}}$ oe	M1
		$\frac{3}{5}x^{\frac{5}{3}} - 9x$ oe	A1
	$= \{\pi\}\left(\left(\frac{3}{5}(125)^{\frac{5}{3}} - 9(125)\right) - \left(\frac{3}{5}(27)^{\frac{5}{3}} - 9(27)\right)\right)$	Substitutes limits of 125 and 27 into an integrated function and subtracts the correct way round.	dM1
	$= \{\pi\}\left((1875 - 1125) - (145.8 - 243)\right)$		
	$= \frac{4236\pi}{5}$ or 847.2π	$\frac{4236\pi}{5}$ or 847.2π	A1
			[5] **12**

Notes for Question 7	
(a)	**B1:** At least one of $\frac{dx}{dt}$ or $\frac{dy}{dt}$ correct. **Note:** that this mark can be implied from their working. **B1:** Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are correct. **Note:** that this mark can be implied from their working. **M1:** Applies their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$, where both $\frac{dy}{dt}$ and $\frac{dx}{dt}$ are trigonometric functions of t. **A1:** $\frac{4}{72}$ or any equivalent correct rational answer not involving surds. Allow $0.0\dot{5}$ with the recurring symbol.

Notes for Question 7 Continued	
	Note: Please check that their $\frac{dx}{dt}$ is differentiated correctly. Eg. Note that $x = 27\sec^3 t = 27(\cos t)^{-3} \Rightarrow \frac{dx}{dt} = -81(\cos t)^{-2}(-\sin t)$ is correct.
(b)	**M1:** Either: • Applying a correct trigonometric identity (usually $1 + \tan^2 t = \sec^2 t$) to give a Cartesian equation in x and y only. • Starting from the RHS and goes on to achieve $\sqrt{9\tan^2 t}$ by using a correct trigonometric identity. • Starts from the LHS and goes on to achieve $\sqrt{9\sec^2 t - 9}$ by using a correct trigonometric identity. **A1*:** For a correct proof of $y = \left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}}$. **Note** this result is printed on the Question Paper, so no incorrect working is allowed. **B1:** Both $a = 27$ and $b = 216$. **Note** that $27 \leqslant x \leqslant 216$ is also fine for B1.
(c)	**B1:** For a correct statement of $\pi\int\left(\left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}}\right)^2$ or $\pi\int\left(x^{\frac{2}{3}} - 9\right)$. Ignore limits and dx. Can be implied. **M1:** Either integrates to give $\pm Ax^{\frac{5}{3}} \pm Bx$, $A \neq 0$, $B \neq 0$ or integrates $x^{\frac{2}{3}}$ correctly to give $\frac{3}{5}x^{\frac{5}{3}}$ oe **A1:** $\frac{3}{5}x^{\frac{5}{3}} - 9x$ or. $\frac{x^{\frac{5}{3}}}{\left(\frac{5}{3}\right)} - 9x$ oe. **dM1:** Substitutes limits of 125 and 27 into an integrated function and subtracts the correct way round. **Note:** that this mark is dependent upon the previous method mark being awarded. **A1:** A correct exact answer of $\frac{4236\pi}{5}$ or 847.2π. **Note:** The π in the volume formula is only required for the B1 mark and the final A1 mark. **Note:** A decimal answer of 2661.557... without a correct exact answer is A0. **Note:** If a candidate gains the first B1M1A1 and then writes down 2661 or awrt 2662 with no method for substituting limits of 125 and 27, then award the final M1A0.
(a) **Way 2**	***Alternative response using the Cartesian equation in part (a)*** $\left\{y = \left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}} \Rightarrow\right\} \frac{dy}{dx} = \frac{1}{2}\left(x^{\frac{2}{3}} - 9\right)^{-\frac{1}{2}}\left(\frac{2}{3}x^{-\frac{1}{3}}\right)$ — $\frac{dy}{dx} = \pm K x^{-\frac{1}{3}}\left(x^{\frac{2}{3}} - 9\right)^{-\frac{1}{2}}$ **M1**; $\frac{dy}{dx} = \frac{1}{2}\left(x^{\frac{2}{3}} - 9\right)^{-\frac{1}{2}}\left(\frac{2}{3}x^{-\frac{1}{3}}\right)$ oe **A1** At $t = \frac{\pi}{6}$, $x = 27\sec^3\left(\frac{\pi}{6}\right) = 24\sqrt{3}$ $\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(\left(24\sqrt{3}\right)^{\frac{2}{3}} - 9\right)^{-\frac{1}{2}}\left(\frac{2}{3}\left(24\sqrt{3}\right)^{-\frac{1}{3}}\right)$ — Uses $t = \frac{\pi}{6}$ to find x and substitutes their x into an expression for $\frac{dy}{dx}$. **dM1** So, $\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(\frac{1}{\sqrt{3}}\right)\left(\frac{1}{3\sqrt{3}}\right) = \frac{1}{18}$ — $\frac{1}{18}$ **A1 cao cso** **Note:** Way 2 is marked as M1 A1 dM1 A1 **Note:** For way 2 the second M1 mark is dependent on the first M1 being gained.

Notes for Question 7 Continued		
7. (b) Way 2	***Alternative responses for M1A1 in part (b)*: STARTING FROM THE RHS**	
	$\{\text{RHS} =\}\ \left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}} = \sqrt{\left(27\sec^3 t\right)^{\frac{2}{3}} - 9} = \sqrt{9\sec^2 t - 9} = \sqrt{9\tan^2 t}$ — For applying $1 + \tan^2 t = \sec^2 t$ oe to achieve $\sqrt{9\tan^2 t}$	M1
	$= 3\tan t = y\ \{= \text{LHS}\}$ **cso** — Correct proof from $\left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}}$ to y.	A1*
	M1: Starts from the RHS and goes on to achieve $\sqrt{9\tan^2 t}$ by using a correct trigonometric identity.	
7. (b) Way 3	***Alternative responses for M1A1 in part (b)*: STARTING FROM THE LHS**	
	$\{\text{LHS} =\}\ y = 3\tan t = \sqrt{\left(9\tan^2 t\right)} = \sqrt{9\sec^2 t - 9}$ — For applying $1 + \tan^2 t = \sec^2 t$ oe to achieve $\sqrt{9\sec^2 t - 9}$	M1
	$= \sqrt{9\left(\frac{x}{27}\right)^{\frac{2}{3}} - 9} = \sqrt{9\left(\frac{x^{\frac{2}{3}}}{9}\right) - 9} = \left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}}$ **cso** — Correct proof from y to $\left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}}$.	A1*
	M1: Starts from the LHS and goes on to achieve $\sqrt{9\sec^2 t - 9}$ by using a correct trigonometric identity.	
7. (c) Way 2	***Alternative response for part (c) using parametric integration***	
	$V = \pi\int 9\tan^2 t\left(81\sec^2 t\sec t\tan t\right)\,\mathrm{d}t$ — $\pi\int 3\tan t\left(81\sec^2 t\sec t\tan t\right)\,\mathrm{d}t$. Ignore limits and $\mathrm{d}x$. Can be implied.	B1
	$= \{\pi\}\int 729\sec^2 t\tan^2 t\sec t\tan t\ \mathrm{d}t$	
	$= \{\pi\}\int 729\sec^2 t\left(\sec^2 t - 1\right)\sec t\tan t\ \mathrm{d}t$	
	$= \{\pi\}\int 729\left(\sec^4 t - \sec^2 t\right)\sec t\tan t\ \mathrm{d}t$	
	$= \{\pi\}\int 729\left(\sec^4 t - \sec^2 t\right)\sec t\tan t\ \mathrm{d}t$	
	$= \{\pi\}\left[729\left(\frac{1}{5}\sec^5 t - \frac{1}{3}\sec^3 t\right)\right]$ — $\pm A\sec^5 t \pm B\sec^3 t$	M1
	$729\left(\frac{1}{5}\sec^5 t - \frac{1}{3}\sec^3 t\right)$	A1
	$V = \{\pi\}\left[729\left(\frac{1}{5}\left(\frac{5}{3}\right)^5 - \frac{1}{3}\left(\frac{5}{3}\right)^3\right) - 729\left(\frac{1}{5}1^5 - \frac{1}{3}1^3\right)\right]$ — Substitutes $\sec t = \frac{5}{3}$ and $\sec t = 1$ into an integrated function and subtracts the correct way round.	dM1
	$= 729\pi\left[\left(\frac{250}{243}\right) - \left(-\frac{2}{15}\right)\right]$	
	$= \frac{4236\pi}{5}$ or 847.2π — $\frac{4236\pi}{5}$ or 847.2π	A1
		[5]

Question Number	Scheme		Marks
8.	$\dfrac{dx}{dt} = k(M - x)$, where M is a constant		
(a)	$\dfrac{dx}{dt}$ is the rate of increase of the mass of waste products.	Any one correct explanation.	B1
	M is the total mass of unburned fuel and waste fuel (or the initial mass of unburned fuel)	Both explanations are correct.	B1
			[2]
(b)	$\int \dfrac{1}{M-x}\,dx = \int k\,dt$ or $\int \dfrac{1}{k(M-x)}\,dx = \int dt$		B1
	$-\ln(M - x) = kt \ \{+ c\}$ or $-\dfrac{1}{k}\ln(M - x) = t \ \{+ c\}$	See notes	M1 A1
	$\{t = 0,\ x = 0 \Rightarrow\}\ -\ln(M - 0) = k(0) + c$	See notes	M1
	$c = -\ln M \Rightarrow -\ln(M - x) = kt - \ln M$		
	then either... $-kt = \ln(M - x) - \ln M$; $-kt = \ln\left(\dfrac{M-x}{M}\right)$; $e^{-kt} = \dfrac{M-x}{M}$	***or...*** $kt = \ln M - \ln(M - x)$; $kt = \ln\left(\dfrac{M}{M-x}\right)$; $e^{kt} = \dfrac{M}{M-x}$	ddM1
	$Me^{-kt} = M - x$	$(M - x)e^{kt} = M$; $M - x = Me^{-kt}$	
	leading to $x = M - Me^{-kt}$ or $x = M(1 - e^{-kt})$ oe		A1 * **cso**
			[6]
(c)	$\left\{x = \dfrac{1}{2}M,\ t = \ln 4 \Rightarrow\right\}\ \dfrac{1}{2}M = M(1 - e^{-k\ln 4})$		M1
	$\Rightarrow \dfrac{1}{2} = 1 - e^{-k\ln 4} \Rightarrow e^{-k\ln 4} = \dfrac{1}{2} \Rightarrow -k\ln 4 = -\ln 2$		
	So $k = \dfrac{1}{2}$		A1
	$x = M\left(1 - e^{-\frac{1}{2}\ln 9}\right)$		dM1
	$x = \dfrac{2}{3}M$	$x = \dfrac{2}{3}M$	A1 **cso**
			[4] 12

Notes for Question 8 Continued	
8. (a)	**B1:** At least one explanation correct. **B1:** Both explanations are correct. $\frac{dx}{dt}$ is the rate of increase of the mass of waste products. or the rate of change of the mass of waste products. M is the total mass of unburned fuel and waste fuel or the initial mass of unburned fuel or the total mass of rocket fuel and waste fuel or the initial mass of rocket fuel or the initial mass of fuel or the total mass of waste and unburned products.
(b)	**B1:** Separates variables as shown. dx and dt should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs. **M1: Both** $\pm\lambda\ln(M-x)$ or $\pm\lambda\ln(x-M)$ **and** $\pm\mu t$ where λ and μ are any constants. **A1:** For $-\ln(M-x)=kt$ or $-\ln(x-M)=kt$ or $-\frac{1}{k}\ln(M-x)=t$ or $-\frac{1}{k}\ln(x-M)=t$ or $-\frac{1}{k}\ln(kM-kx)=t$ or $-\frac{1}{k}\ln(kx-kM)=t$ **Note:** $+c$ is not needed for this mark. **IMPORTANT:** $+c$ can be on either side of their equation for the 1st A1 mark. **M1:** Substitutes $t=0$ AND $x=0$ in an integrated or changed equation containing c (or A or $\ln A$, etc.) **Note** that this mark can be implied by the correct value of c. **ddM1:** Uses their value of c which must be a ln term, and uses fully correct method to eliminate their logarithms. **Note:** This mark is dependent on both previous method marks being awarded. **A1:** $x=M-Me^{-kt}$ or $x=M(1-e^{-kt})$ or $x=\frac{M(e^{kt}-1)}{e^{kt}}$ or equivalent where x is the subject. **Note:** Please check their working as incorrect working can lead to a correct answer. **Note:** $\left\{\frac{dx}{dt}=k(M-x)\Rightarrow\frac{dx}{dt}=\frac{1}{kM-kx}\Rightarrow\right\}x=-\frac{1}{k}\ln(kM-kx)\{+c\}$ is B1(Implied) M1A1.
(c)	**M1:** Substitutes $x=\frac{1}{2}M$ and $t=\ln 4$ into one of their earlier equations connecting x and t. **A1:** $k=\frac{1}{2}$, which can be an un-simplified equivalent numerical value. i.e. $k=\frac{\ln 2}{\ln 4}$ is fine for A1. **dM1:** Substitutes $t=\ln 4$ and their evaluated k (which must be a numerical value) into one of their earlier equations connecting x and t. **Note:** that the 2nd Method mark is dependent on the 1st Method mark being awarded in part (c). **A1:** $x=\frac{2}{3}M$ **cso**. **Note:** Please check their working as incorrect working can lead to a correct answer.

Notes for Question 8 Continued		
***Aliter* 8. (b) Way 2**	$\int \frac{1}{M-x}\,dx = \int k\,dt$	B1
	$-\ln(M-x) = kt \ \{+c\}$ See notes	M1 A1
	$\ln(M-x) = -kt + c$	
	$M - x = Ae^{-kt}$	
	$\{t = 0,\ x = 0 \Rightarrow\}\ M - 0 = Ae^{-k(0)}$	M1
	$\Rightarrow M = A$	
	$M - x = Me^{-kt}$	ddM1
	So, $x = M - Me^{-kt}$	A1
		[6]
(b)	**B1M1A1:** Mark as in the original scheme. **M1:** Substitutes $t = 0$ AND $x = 0$ in an integrated equation containing their constant of integration which could be c or A. **Note** that this mark can be implied by the correct value of c or A. **ddM1:** Uses a fully correct method to eliminate their logarithms and writes down an equation containing their evaluated constant of integration. **Note:** This mark is dependent on both previous method marks being awarded. **Note:** $\ln(M-x) = -kt + c$ leading to $\ln(M-x) = e^{-kt} + e^{c}$ or $\ln(M-x) = e^{-kt} + A$ would be dddM0. **A1:** Same as the original scheme.	
***Aliter* 8. (b) Way 3**	$\int_0^x \frac{1}{M-x}\,dx = \int_0^t k\,dt$	B1
	$\left[-\ln(M-x)\right]_0^x = \left[kt\right]_0^t$	M1 A1
	$-\ln(M-x) - (-\ln M) = kt$ Applies limits of	M1
	$-\ln(M-x) + \ln M = kt$	
	and then follows the original scheme.	
(a)	**B1M1A1:** Mark as in the original scheme (ignoring the limits). **ddM1:** Applies limits 0 and x on their integrated LHS and limits of 0 and t. **M1A1:** Same as the original scheme.	

Notes for Question 8 Continued		
Aliter 8. (b) Way 4	$\int \frac{1}{M-x}\,\mathrm{d}x = \int k\,\mathrm{d}t \quad \left\{\Rightarrow \int \frac{-1}{x-M}\,\mathrm{d}x = \int k\,\mathrm{d}t\right\}$	B1
	$-\ln\lvert x-M\rvert = kt + c$ — *Modulus not required for 1st A1.*	M1 A1
	$\{t=0,\ x=0 \Rightarrow\}\ -\ln\lvert 0-M\rvert = k(0)+c$ — *Modulus not required here!*	M1
	$\Rightarrow c = -\ln M \ \Rightarrow -\ln\lvert x-M\rvert = kt - \ln M$	
	then either... $-kt = \ln\lvert x-M\rvert - \ln M$; $-kt = \ln\left\lvert\frac{x-M}{M}\right\rvert$ ***or...*** $kt = \ln M - \ln\lvert x-M\rvert$; $kt = \ln\left\lvert\frac{M}{x-M}\right\rvert$	
	As $x < M$	
	either: $-kt = \ln\left(\frac{M-x}{M}\right)$; $\mathrm{e}^{-kt} = \frac{M-x}{M}$ *or:* $kt = \ln\left(\frac{M}{M-x}\right)$; $\mathrm{e}^{kt} = \frac{M}{M-x}$ — *Understanding of modulus is required here!*	**ddM1**
	either: $M\mathrm{e}^{-kt} = M - x$ *or:* $(M-x)\mathrm{e}^{kt} = M$; $M - x = M\mathrm{e}^{-kt}$ leading to $x = M - M\mathrm{e}^{-kt}$ or $x = M(1-\mathrm{e}^{-kt})$ oe	**A1 * cso**
		[6]
	B1: Mark as in the original scheme. **M1A1M1:** Mark as in the original scheme ignoring the modulus. **ddM1:** Mark as in the original scheme **AND** the candidate must demonstrate that they have converted $\ln\lvert x-M\rvert$ to $\ln(M-x)$ in their working. **Note:** This mark is dependent on both the previous method marks being awarded. **A1:** Mark as in the original scheme.	
Aliter 8. (b) Way 5	***Use of an integrating factor*** **(I.F.)**	
	$\frac{\mathrm{d}x}{\mathrm{d}t} = k(M-x) \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}t} + kx = kM$	
	I.F. $= \mathrm{e}^{kt}$	B1
	$\frac{\mathrm{d}}{\mathrm{d}t}\left(\mathrm{e}^{kt}x\right) = kM\mathrm{e}^{kt}$,	
	$\mathrm{e}^{kt}x = M\mathrm{e}^{kt} + c$	M1A1
	$x = M + c\mathrm{e}^{-kt}$	
	$\{t=0,\ x=0 \Rightarrow\}\ 0 = M + c\mathrm{e}^{-k(0)}$	M1
	$\Rightarrow c = -M$	
	$x = M - M\mathrm{e}^{-kt}$	ddM1A1

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